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Foundations of Query Languages
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10. Exercise Set: Assorted

Exercise 1
Consider the inductive definition of the quantifier rank (qr) of a formula:

$$\begin{aligned} \mathrm{qr} &: \text{Formulae} \to \mathbb{N} \\ \mathrm{qr}(t_1 = t_2) &:= 0 \\ \mathrm{qr}(R(t_1, \ldots, t_n)) &:= 0 \\ \mathrm{qr}(\phi \wedge \psi) &:= \max(\mathrm{qr}(\phi), \mathrm{qr}(\psi)) \\ \mathrm{qr}(\neg\phi) &:= \mathrm{qr}(\phi) \\ \mathrm{qr}(\exists x\ \phi) &:= \mathrm{qr}(\phi) + 1 \end{aligned}$$

a) Give an intuition of what the quantifier rank of a first-order formula is.
b) Compute $\mathrm{qr}\big(\exists x_1 \exists x_2 (x_1 = x_2 \wedge \neg\exists x_3\ R(x_1, x_2, x_3))\big)$ and $\mathrm{qr}\big(\exists x_1\ (\exists x_2\ x_1 = x_2 \wedge \neg\exists x_3\ S(x_1, x_3))\big)$.
c) Give two first-order formulas φ, ψ such that $\varphi \equiv \psi$, but $\mathrm{qr}(\varphi) \neq \mathrm{qr}(\psi)$.

Exercise 2
Consider the following constant-free Datalog program:

$$\begin{aligned} &P(x) \leftarrow P_0(x) \\ &P(x) \leftarrow R(x, y), P(y) \\ &R(x, y) \leftarrow S(x), S(y). \end{aligned}$$

a) Compute the set of extensional and the set of intensional relations.
b) Is the program bounded? If so, prove your claim and give an equivalent non-recursive Datalog program, otherwise give a counterexample.

Exercise 3
Compute the dependency graphs for the following Datalog programs:

- $P(x) \leftarrow P_0(x)$
  $P(x) \leftarrow R(x,y), P(y)$
  $R(x,y) \leftarrow S(x), S(y)$

- $greenPath(x,y) \leftarrow green(x,y)$
  $greenPath(x,y) \leftarrow greenPath(x,z), greenPath(z,y)$
  $bingo(x,y) \leftarrow red(x,y), \neg greenPath(x,y)$

- $win(x) \leftarrow move(x,y), \neg win(y)$

Which of these programs are recursive?

Exercise 4
Let the EDB relations $visits(guest, bar)$, $serves(bar, beer)$, and $likes(guest, beer)$ be given. The first relation indicates which guests visit which bar. The second relation contains information on which beers can be purchaesd in which bar. The third relation stores information on ehat guest likes what beer.

You may assume that every guest visits at least one bar and likes at least one kind of beer.

Define the following predicates in Datalog:

- $content(G)$ holds if a guest G visits a bar which serves a beer that he likes.
- $recommendation(G, B)$ holds if the guest G can purchase a beer that he likes in the bar B.
- $frustrated(G)$ holds if guest G does not visit a bar in which he can purchase a beer that he likes.

Due by: July 7, 2010 before the tutorial starts.